

UNIT 1: REAL NUMBERS, POWERS AND ROOTS

Exercise 1: Classify the following numbers:

- a) π ; $7/5$; $0.477777\dots$; $-\sqrt{9}$; -2 ; $\sqrt{3}$; $\sqrt{-25}$; $14/7$; $\sqrt[3]{-8}$
 b) $\sqrt{-4}$; $7\bar{2}$; $-\sqrt{16}$; $20/7$; -5 ; $\sqrt{2}$; $21/3$; $-\sqrt[3]{-27}$; ϕ

Exercise 2: Decide if the following affirmations are true or false and why

- a) All rational numbers are integer numbers.
 b) Irrational numbers are not real numbers.
 c) The product of two rational numbers can never be a natural number.
 d) The sum of two irrational numbers is always an irrational number.

Exercise 3: Decide if the following affirmations are true or false and why

- a) The quotient of two rational numbers can never be a natural number.
 b) Rational numbers are natural numbers.
 c) Any power of an irrational number is always an irrational number.
 d) Some numbers are neither rational nor irrational numbers.

Exercise 4: Represent $\sqrt{34}$ on the number line using just a compass and a ruler

Exercise 5: Represent $\sqrt{41}$ on the number line using just a compass and a ruler

Exercise 6: Susan uses the intercept theorem to measure the church tower and gets a result of 19.8m. Then she takes a look at the church records and finds out that the real height is of 20.1m. find the percentage error between both measures.

Exercise 7: A racing car is said to reach a speed of 320 km/h, but when tested several times on a track, it only reached 307.6 km/h. Find the percentage error between the car specifications and the real value.

Exercise 8: A realtor bought a house for a price of 135000€ and sold it for 150000€, while a second realtor bought a house for 225000€ and sold it for 240000€. If the real state agency has a policy saying that the benefit must be, at least, 10% of the purchase price, which realtor is going to be fired?

Exercise 9: Find the absolute and relative errors when we approximate ϕ by $\frac{987}{610}$

Exercise 10: Round and chop the number $e \approx 2.7182818285$ to five significant figures and estimate both the absolute and relative errors. Which approximation is better? Why?

Exercise 11: A few years ago I went to the butcher's shop to get some croquettes and skewer meat and the bill added up to a total of 6.67€. The butcher asked me to pay 6.70€. Find the percentage of money that I was cheated. Do you think the approximation is appropriate?

Exercise 12: Write the following intervals as inequalities and represent them on the number line:

- | | | |
|---------------------|-------------------|-------------------------|
| a) $[-10, +\infty)$ | b) $(-2, 6]$ | c) $(-9, -2)$ |
| d) $[0, 7]$ | e) $(-\infty, 2]$ | f) $(-\infty, +\infty)$ |

Exercise 13: Write the following inequalities in interval notation and then represent them on the number line.

- | | | |
|-------------------|--------------------|----------------------|
| a) $4 \leq x < 9$ | b) $-3 < x \leq 4$ | c) $-3 \leq x$ |
| d) $-2 < x < -7$ | e) $x < 5$ | f) $0 \leq x \leq 1$ |

Exercise 14: Write as an interval:

- | | | |
|----------------------------|----------------------------|--------------------------|
| a) $(-7, 3) \cup [-3, 1]$ | b) $(-7, -2) \cup [-2, 3]$ | c) $[-5, 2) \cap (0, 3]$ |
| d) $(-9, -3) \cup (-3, 1]$ | e) $[-3, 1] \cap [1, 4]$ | f) $[-4, 5) \cap (0, 3]$ |

Exercise 15: Study these unions and intersections of intervals:

- | | | |
|--------------------------|--------------------------|-------------------------|
| a) $(-3, 1) \cap (0, 4)$ | b) $(-4, 7) \cup [3, 5)$ | c) $(1, 5) \cup [5, 9]$ |
|--------------------------|--------------------------|-------------------------|

Exercise 16: Write the following numbers using scientific notation:

- $314159265358979323846264338327950 =$
- $0.000000027182818285 =$
- $853.794 \cdot 10^{-5} =$
- $0.0032864 \cdot 10^7 =$
- $42835.729 \cdot 10^4 =$
- $4672314.25 \cdot 10^{-9} =$
- $0.00016234 \cdot 10^{-7} =$
- $345.7865 \cdot 10^4 =$

Exercise 17: Write the following numbers using scientific notation:

- Jupiter's mass: 1898 200 000 000 000 000 000 000 000 kg
- The radius of an atom of hydrogen: 0.000 000 000 052 917 721 m
- The mass of a dust particle 0.000000000753 kg
- The distance between the Sun and Pluto 5906380000 km

Exercise 18: The Atlantic Ocean expands at a speed of $6.34 \cdot 10^{-10}$ m/s. How much does the distance between Europe and America grow in a year?

Exercise 19: The New Horizons probe can reach a speed of 58536 km/h. How long would it take it to cover the distance between the Earth and Jupiter, which is estimated in $5.88 \cdot 10^{11}$ m?

Exercise 20: Work out:

- a) $5.12 \cdot 10^2 - 4.37 \cdot 10^5 - 1.83 \cdot 10^7 =$
 b) $-3.51 \cdot 10^{-2} + 7.92 \cdot 10^{-3} - 5.84 \cdot 10^{-6} =$
 c) $2.51 \cdot 10^4 - 7.43 \cdot 10^5 - 8.31 \cdot 10^7 =$
 d) $5.84 \cdot 10^{-5} - 5.13 \cdot 10^{-3} + 7.9 \cdot 10^{-6} =$

Exercise 21: Work out:

- a) $(7.35 \cdot 10^4) \cdot (7.15 \cdot 10^{-5}) =$
 b) $(4.12 \cdot 10^{-9}) \cdot (7.35 \cdot 10^4) =$
 c) $(4.48 \cdot 10^3) : (9.05 \cdot 10^{-8}) =$
 d) $(3.72 \cdot 10^{-9}) : (9.4 \cdot 10^{-2}) =$
 e) $2.39 \cdot 10^5 - 5.64 \cdot 10^2 : 7.15 \cdot 10^{-2} =$
 f) $4.12 \cdot 10^{-3} + 8.29 \cdot 10^5 \cdot 7.34 \cdot 10^{-7} =$

Exercise 22: Work out the value of the following roots:

- a) $\sqrt{151200} =$ b) $\sqrt[3]{1024} =$ c) $\sqrt[5]{291600000} =$
 d) $\sqrt[12]{\frac{x^{20}y^{36}z^4}{w^{30}}} =$ e) $\sqrt{\frac{7^4}{5^3}} =$ f) $\sqrt[7]{\frac{2^{14} \cdot 3^{21} \cdot 7^{35}}{5^4 \cdot 11^{48}}} =$
 g) $\sqrt[3]{625000} =$ h) $\sqrt[5]{\frac{a^{-10} \cdot b^{25}}{c^{17}}} =$ i) $\sqrt[7]{\frac{x^{16} \cdot y^{-39} \cdot z^{-12}}{w^7}} =$

Exercise 23: Work out:

- a) $\sqrt{300} - 5\sqrt{27} + 7\sqrt{48} =$
 b) $2\sqrt{32} - 4\sqrt{243} + 5\sqrt{243} + \sqrt{75} =$
 c) $3\sqrt{108} - 9\sqrt{128} - \sqrt{75} + \sqrt{2} =$
 d) $\sqrt{891} - 2\sqrt{1323} + 7\sqrt{2187} - \sqrt{275} + 5\sqrt{300} =$
 e) $5\sqrt{162} - 2\sqrt{175} + 5\sqrt{128} - \sqrt{343} =$

Exercise 24: Work out:

- a) $\sqrt{3} \cdot \sqrt[3]{3} \cdot \sqrt[4]{3} =$ b) $\frac{\sqrt[4]{3^5 \cdot 5^7} \cdot \sqrt[3]{3^2 \cdot 5}}{\sqrt{3 \cdot 5^3}} =$
 c) $\frac{\sqrt{2^4 \cdot 5^3} \cdot \sqrt[5]{2 \cdot 3^7}}{\sqrt[3]{2^4 \cdot 5}} =$ d) $\frac{\sqrt{2^{-3} \cdot 5^7} \cdot \sqrt[3]{7^2}}{\sqrt[5]{2^{-4} \cdot 7^6}} =$
 e) $\frac{\sqrt[6]{2^5 \cdot 7^3} \cdot \sqrt[5]{7^4 \cdot 5^3}}{\sqrt{2 \cdot 5^{-2}}} =$ f) $\frac{\sqrt[3]{3^2 \cdot 5^3} \cdot \sqrt[4]{7^3 \cdot 3^5}}{\sqrt{5^4 \cdot 3^{15}}} =$

Exercise 25: Express as a radical:

a) $5^{2/3} =$

b) $7^{5/2} =$

c) $3^{-2/5} =$

d) $x^{-4/7} =$

e) $y^{11/6} =$

f) $a^{-5/3} =$

Exercise 26: Work out (you can either work with the radicals or transform them into rational exponents)

a) $\sqrt{\sqrt{\sqrt{a^{10}}}} =$

b) $(\sqrt[7]{a^3})^5 =$

c) $\sqrt[3]{\sqrt[4]{y^{12}}} =$

d) $(\sqrt[6]{\sqrt{a^{-5}}})^8 =$

e) $(\sqrt{\sqrt[4]{x^3 y^5}})^{10} =$

e) $(\left(\left(\left(\sqrt{\sqrt[6]{a^2 b^{-3}}}\right)\right)\right))^8 =$

Exercise 27: Work out and express as a single radical:

a) $\sqrt{a} : \sqrt[5]{a} =$

b) $\sqrt{x^3} \cdot \sqrt[3]{x^{-4}} \cdot \sqrt[4]{x^{-5}} =$

c) $\sqrt{a^3} \cdot \sqrt[3]{a^4} \cdot \sqrt[4]{a^5} : \sqrt[5]{a^6} =$

d) $(\sqrt[15]{a^2} : \sqrt{a^{-3}}) : \sqrt[4]{a^7} =$

e) $\frac{\sqrt[3]{x^5} \cdot \sqrt{x^{-7}}}{\sqrt[5]{x^{-8}}} =$

f) $\frac{\sqrt[5]{y^2} \cdot \sqrt{y^{-7}}}{\sqrt[3]{y^{-4}} \cdot \sqrt[10]{y^3}} =$

Exercise 28: Work out and express as a radical:

a) $7^{-3/5} \cdot 7^{2/3} \cdot 7^{1/4} \cdot 7^{-3/2} =$

b) $a \cdot a^{7/3} \cdot a^{-2/5} \cdot a^{-1/2} =$

c) $x^{-2/5} \cdot x^{6/7} \cdot x^{1/10} \cdot x^{2/3} \cdot x^2 =$

d) $3^{2/5} \cdot 3^{3/2} \cdot 3^{5/6} \cdot 3^{1/10} =$

e) $y^{7/5} \cdot y^{2/3} \cdot y^{-5/4} \cdot y^{-3/5} =$

Exercise 29: Rationalize and simplify if possible:

a) $\frac{5}{\sqrt{5}} =$

b) $\frac{7\sqrt{3}}{\sqrt{7}} =$

c) $\frac{5}{\sqrt{10}} =$

d) $\frac{7}{\sqrt{14}} =$

e) $\frac{1}{\sqrt[3]{5^2}} =$

f) $\frac{6}{\sqrt[4]{3}} =$

g) $\frac{10\sqrt[3]{2}}{\sqrt[2]{2^3}} =$

h) $\frac{21}{\sqrt[5]{7^4}} =$

Exercise 30: Rationalize and simplify if possible:

a) $\frac{7}{4-\sqrt{3}} =$

c) $\frac{3}{4+\sqrt{7}} =$

e) $\frac{15}{\sqrt{7}-\sqrt{2}} =$

b) $\frac{5}{\sqrt{3}+\sqrt{2}} =$

d) $\frac{2\sqrt{7}}{\sqrt{5}-\sqrt{3}} =$

f) $\frac{14}{3-\sqrt{2}} =$

Exercise 31: Rationalize and simplify if possible:

a) $\frac{\sqrt{3}+1}{\sqrt{3}-1} =$

c) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}+\sqrt{2}} =$

e) $\frac{\sqrt{2}-\sqrt{10}}{\sqrt{2}+\sqrt{10}} =$

b) $\frac{3-\sqrt{5}}{3+\sqrt{5}} =$

d) $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} =$