

## UNIT 9: PROBABILITY

### **CLASS EXERCISES:**

- C1) Find the sample space of the experiment throwing a die.
- C2) Let's throw a die. Write all the possible outcomes associated to the events:
- We get 1
  - We get an even number
  - We get less than 3
  - We get more than 6
  - We get a number between 1 and 6
- C3) Write the sample space of the experiment throwing a coin twice
- C4) We have three coins. Two of them are normal coins but the third one has two tails. Write the sample space of the experiment throwing the three coins
- C5) Write the complement of the events of the second example
- C6) Given the events  $A = \{2, 4, 6, 8, 10\}$ ,  $B = \{1, 2, 3, 4, 5\}$ ,  $C = \{3\}$  and  $D = \{14\}$ , find:
- $A \cup B$
  - $B \cup C$
  - $B \cap C$
  - $B \cup D$
  - $A \cap B$
  - $A \cap C$
- C7) Find the probability of these events when throwing a die:
- $A \equiv$  get a prime number
  - $B \equiv$  get a two
  - $C \equiv$  get more than four
- C8) Find the probability of these events when drawing a card from a Spanish deck of cards:
- $P(\text{drawing a cup})$
  - $P(\text{drawing a face card})$
  - $P(\text{drawing the jack of spades})$
  - $P(\text{not drawing a coin card})$
  - $P(\text{drawing a joker})$
  - $P(\text{drawing a cup, coin, club or spade card})$

- C9) In an Erasmus+ Project there are 8 Spanish people, 11 Greek people, 9 Turkish people and 3 kangaroos. Taking a random individual find the probability of the events:
- They are Spanish
  - They are not Spanish
  - They belong to the European Union
  - They are not human
  - They are from Africa
- C10) If  $P(\bar{A}) = 0.6$ ,  $P(B) = 0.2$  and  $P(A \cap B) = 0.1$ , find:
- $P(A) =$
  - $P(A \cup B) =$
- C11) Let's consider a Spanish deck of cards. We get two cards successively with replacement. Find the probability of the events:
- We get two gold cards
  - The first is a face card and the second is a cup
  - We don't get an ace
  - We get a face card and a cup
- C12) In a fair shooting attraction the probability of hitting the bull's eye is 0.3. Find the probability of the events:
- I hit the bull's eye three times in a row
  - I hit the bull's eye two out of the three times
  - I hit the bull's eye at least once
- C13) If  $P(A) = 0.6$ ,  $P(\bar{B}) = 0.2$  and  $P(A \cup B) = 0.9$  find  $P(B / A)$
- C14) We draw two cards from a Spanish set of cards without replacement. What's the probability that the first one is a king and the second one is an ace?
- C15) An urn contains 4 black balls, 3 white balls and 2 red balls. I draw three balls without replacement. Find the probability that:
- All three balls are white
  - All three balls are black
  - A white ball and two black ones
  - No red balls
  - At least one red ball
- C16) An urn contains 4 white balls and 2 black balls. We draw two balls without replacement.
- Find the probability that both balls are white
  - If the first ball is black, what's the probability that the second one is also black?
- C17) Given two events so that  $P(A) = 0.4$ ,  $P(\bar{B}) = 0.25$  and  $P(B / A) = 0.7$ , find
- $P(A \cup B)$
  - $P(A / B)$
  - $P(\overline{A \cap B}) =$
  - Are  $A$  and  $B$  independent events? Are they mutually exclusive?

**C18) In a class we have 25 students, 17 of which are girls. 2 boys and three girls are left-handed. Taking a random student from that class find the probability that:**

- a) They are left-handed**
- b) He's a boy, given that he's not left-handed**

**C19) In a certain edition of the Mediterranean Games, 60% of the athletes were European, 25% were American and the rest were African. 40% of the European participants, 25% of the Americans and 10% of the African can speak Spanish. Taken a random athlete find the probability that:**

- a) They are American and they can speak Spanish**
- b) They speak Spanish**
- c) They're European knowing that they can't speak Spanish**

**C20) There are 50 teachers in a certain high school and 28 of them teach Science subjects. 21 of the Science teachers and 18 of the Humanity ones have a permanent position. Taken a random teacher find the probability that:**

- a) They don't have a permanent position**
- b) They teach Humanity subjects and they have a permanent position**
- c) They teach Science given that they have a permanent position**

**C21) When it snows, 47% of the kids in the city go sledging, 32% go skiing and the rest stay home. The probability of having an accident while sledging is 23%, while skiing 17% and if staying home 2%. Taking a random person find the probability that:**

- a) They went skiing and had an accident.**
- b) They had an accident.**
- c) Knowing that Tony had an accident yesterday, find the probability that he went sledging.**

**Exercise 1:** Given a deck of Spanish playing cards write the outcomes of the events:

- I get an ace
- I get a face card
- I get a spade card
- I don't get an ace
- I don't get either a coin card or a club card
- I get a card with an odd value

**Exercise 2:** We throw a die and let's consider the events

$A$  = getting a prime number

$B$  = getting an odd number

$C$  = getting more than four

Write the following events:

- Sample space
- $A \cup B$ ,  $A \cap B$
- $\bar{A}$  and  $\bar{B}$
- $\bar{A} \cup \bar{B}$  and  $\bar{A} \cap \bar{B}$
- $\overline{A \cup B}$  and  $\overline{A \cap B}$
- $(A \cap B) \cup \bar{C}$

**Exercise 3:** In a class of 25 students, 10 of them play football, 12 play some instrument, 15 attend an academy, 7 attend the academy and play an instrument, and 18 play football or attend an academy. Taking a random student find the probability of the events:

- They play an instrument
- They attend the academy
- They play football
- They attend the academy or play an instrument
- They play football and attend the academy
- They play football and an instrument

**Exercise 4:** We draw two cards from a Spanish deck of cards with replacement.

- What is the probability that both of them will be fives?
- What is the probability that both cards drawn will be spades?
- What is the probability of drawing a king and a seven?

**Exercise 5:** If  $P(A) = 0.3$ ,  $P(\bar{B}) = 0.4$  and  $P(A \cup B) = 0.5$  find  $P(A \cap B)$

**Exercise 6:** Given two random events so that  $P(A) = 0.4$ ,  $P(\bar{B}) = 0.3$  and  $P(A \cup B) = 0.98$ , are they independent?

**Exercise 7:** Given two random events A and B so that  $P(\bar{B}) = 0.4$  and  $P(A \cup B) = 0.7$  find  $P(A)$  so that they are independent

**Exercise 8:** I paint in blue the first three faces of a die (1-2-3), in red the next two faces (4-5) and in green the last one (6). If I throw the die once find the probability that:

- a) I get a red face
- b) I get an odd number
- c) I get less than three
- d) I get a blue face or an even number
- e) I get a red face or less than three
- f) I get a green face and an odd number
- g) I get a red face and an even number

**Exercise 9:** Let's consider the experiment of drawing a card from a Spanish set of cards without replacement. Find the probability that:

- a) We get a spade card
- b) We get a club face card
- c) We get a cups card or a two
- d) We get a coin king

**Exercise 10:** I draw three cards from a Spanish deck of cards without replacement. What's the probability of getting three club cards?

**Exercise 11:** I toss a coin four times. What's the probability of

- a) Getting four tails
- b) Getting at least one tail

**Exercise 12:** I have an urn with 5 white balls, 2 black balls and 3 red balls. I get 3 balls without replacement. Find the probability that:

- a) I get three white balls
- b) The first one is black, the second is red and the third is white
- c) I get a ball of each color
- d) I get at least a red ball

**Exercise 13:** In a certain Spanish city, 45% of its citizens can speak English, 30% can speak French and 15% can speak both languages. Taking a random citizen find the probability that:

- a) They can speak English if they can speak French
- b) They can speak English or French
- c) They can speak neither English nor French

**Exercise 14:** In a certain village we have only two supermarkets A and B. 58% of the citizens buy items in the supermarket A, 35% of them buy them in the supermarket B, and 12% of them buy things in both supermarkets. Taking a random citizen, find the probability that:

- a) They go to any of the supermarkets
- b) They don't go to any of the supermarkets
- c) They go to supermarket A knowing that they go to supermarket B

**Exercise 15:** There are 28 Science teachers in a high school, of which 12 are women. 4 of the woman and 3 of the men are Math teachers. Taken a random teacher find the probability that:

- They teach Math
- They don't teach Math
- She's a woman knowing that they teach Math

**Exercise 16:** There's a 70% probability that tomorrow will be sunny. In that case, the probability that the day after will be sunny too is 60%. The probability of the day after tomorrow being sunny if tomorrow is overcast is 45%. Find the probability that:

- The day after tomorrow will be overcast
- The day after tomorrow will be sunny given that tomorrow is sunny
- Tomorrow and the day after will be cloudy

**Exercise 17:** There are 29 students in a class, 17 of which are girls. During the May Fair, 37% of the girls and 42% of the boys are riding the Ferris Wheel. Taken a random student, find the probability that:

- He's a boy and he's not riding the Ferris Wheel
- She's a girl and she's riding the Ferris wheel
- She is a girl, knowing that they will ride the Ferris Wheel

**Exercise 18:** I have two urns. The first one contains 3 black balls, 2 white ones and 1 red one. The second one contains 4 black balls, 1 white one and 3 red ones.

I throw a die. If I get less than 3, I choose the first one. Otherwise, I choose the second. Find the probability that:

- I get a red ball
- I get a number greater than or equal to 3 and a white ball
- I got less than 3 knowing that I have a black ball in my hand

**Exercise 19:** Nowadays 23% of French parents don't vaccinate their kids against measles. The probability of suffering from measles when in contact with the virus is of 1% if vaccinated and 90% if not. Taken a random French child find the probability that:

- They have suffered from measles
- They weren't vaccinated knowing that they have suffered from measles
- They were vaccinated given that they haven't suffered from measles

**Exercise 20:** 24.2% of homes in Spain are formed by just one person. 82% of the one-person homes and 75% of the more-than-one-person homes recycle cans and bricks. Taken a random home find the probability that:

- They don't recycle
- They are formed by just one person, knowing that they recycle

**Exercise 21:** 24% of Spanish men are smokers and they have a 28% probability of developing a lung cancer. 2% of the non-smokers may also develop a lung cancer. Expressing your answers as percentages, find the probability that:

- A man develops a lung cancer
- He's a smoker, given that he has lung cancer