UNIT 2: POLYNOMIALS AND ALGEBRAIC FRACTIONS

Exercise 1: Given the polynomials:

$$P(x) = 5x^6 - 4x^5 + 3x^2 - 8x + 9$$
$$Q(x) = 7x^6 + 2x^5 - 8x^4 + 5x - 10$$
$$R(x) = -8x^6 + 5x^5 - 7x^3 - 4x^2 + 5x$$

Work out:

a) $P + Q =$ b) $P + R =$

c) $R - Q =$ d) $Q - R =$

Exercise 2: Work out:

a) $(3x^2 + x + 1)(x - 5) =$

b) $(4x^3 - 3x^2 + 7x - 2)(x^2 - 2x) =$

c) $(7x^3y^2 - x^2y + 3x^2y^2)(xy - x) =$

d) $(2x - 7)(2x + 7) =$

e) $(2xz^2 - 5x^2z - z)(x - z) =$

f) $x(x^2 - 7x) - x^2(x - 2) =$

g) $(7x + 2)(7x + 2) =$

h) $(x - 1)(x - 2) =$

i) $(x + 7)(x + 4) =$

Exercise 3: Expand these expressions using quadratic multiplication formulas:

a) $(x + 2)^2 =$ b) $(y - 8)(y + 8) =$

c) $(3x - 5)^2 =$ d) $(5x^2 - 2)^2 =$

e) $(9a^2 - 5)(9a^2 + 5) =$ f) $(x^3 - 5x^2)^2 =$

g) $(\sqrt{2} + \sqrt{3})^2 =$ h) $(4 + 2\sqrt{3})^2 =$

i) $(5\sqrt{2} - 1)^2 =$ j) $(\sqrt{5} - 2\sqrt{7})^2 =$

Exercise 4: Expand these expressions using quadratic multiplication formulas:

a) $(x^3 - y^4)^2 =$ b) $(2u^2 - 5w^7)^2 =$

c) $(5x + 3y)(5x - 3y) =$ d) $(xyz + w^5)(xyz - w^5) =$

e) $(x^2y^3 - w^6)^2 =$ f) $(3x^2y - 4x^3z^5)^2 =$

Exercise 5: Express using quadratic multiplication formulas:

a) $x^2 - 10x + 25 =$

b) $y^4 + 14y^2 + 49 =$

c) $a^2 - 4b^2 =$

d) $25u^2 - 20uv + 4v^2 =$

e) $x^2 - 15xy^3 + 25y^6 =$

f) $z^2 + 8z - 16 =$

g) $36x^{12}y^8z^4 - 60x^6y^4z^2a^2b^5 + 25a^4b^{10} =$

Exercise 6: Divide the following polynomials and indicate the quotient and the remainder:

a) $(4x^3 - 5x^2 + 7):(x-3) =$

b) $(x^4 - 7x^3 + 7x - 1):(x^2 + 2x) =$

c) $(x^5 - x^4 + 5x^2 - x):(x^2 + 2) =$

d) $(x^4 - 16):(x^2 - 4) =$

Exercise 7: Divide the following polynomials and indicate the quotient and the remainder:

a) $(x^3 - x^2 + x):(x^2 - 2) =$

b) $(2x^3 + 7x^2 - 3x):(x-5) =$

c) $(x^6 - 7x^4 + 2x^2 - 5):(x^3 + 3) =$

d) $(x^7 + 1):(x+1) =$

Exercise 8: Divide the following polynomials using Ruffini's rule and indicate the quotient and the remainder:

a) $(x^5 - 4x^3 + 2x^2 - 5x + 2):(x+1)$

b) $(x^5 - 2x^4 + 5x^2 - x + 4):(x+2)$

c) $(x^5 - 7x^3 + 4x^2 - 3):(x-2) =$

d) $(x^3 - 1):(x+1)$

Exercise 9: Divide the following polynomials using Ruffini's rule and indicate the quotient and the remainder:

a) $(x^3 - 10x^2 + 3):(x-2)$

b) $(x^5 - 4x^3 + 5x^2 - x + 4):(x+2)$

c) $(x^5 - 2x^3 + 2x + 1):(x+1)$

d) $(4x^3 + 5x^2 - 10x + 4):(x+3)$

Exercise 10: Given the polynomials

$P(x) = x^4 - 4x^3 + 5x - 7$

$Q(x) = x^2 - 2x$

$R(x) = x + 3$

a) Divide $P(x)$ by $Q(x)$

b) Divide $P(x)$ by $R(x)$

In both cases, indicate the quotient and the remainder

Exercise 11: Check if the polynomial $P(x) = x^{72} - 3x^{48} - 4$ is divisible by $(x+1)$

Exercise 12: Work out the value of k so that when dividing $P(x) = x^4 - kx^3 + 9x^2 - 8x + 3$ by $(x-1)$ the remainder is ten

Exercise 13: Work out the value of k so that $P(x) = kx^3 - 3x^2 + 2x - 5$ is divisible by $(x-2)$

Exercise 14: Factor the following polynomials and indicate their roots:

a) $P(x) = x^3 - 4x^2 + x + 6$

b) $P(x) = x^5 + 3x^4 + 4x^3 + 6x^2 + 4x$

c) $P(x) = x^3 + 5x^2 + 8x + 4$

d) $P(x) = x^5 - 2x^4 - 9x^3 + 18x^2$

e) $P(x) = x^5 - 4x^4 - 26x^3 - 44x^2 - 31x - 8$

f) $P(x) = x^4 + 13x^3 + 33x^2 + 31x + 10$

Exercise 15: Write a polynomial whose roots are $x = -1$ double, $x = \pm 3$

Exercise 16: Solve and factorize the following equations:

a) $x^4 - 10x^2 + 9 = 0$

b) $x^4 - 9x^2 + 20 = 0$

c) $x^4 - 18x^2 + 81 = 0$

d) $x^4 + 13x^2 + 12 = 0$

Exercise 17: Factor the following polynomials and indicate their roots:

a) $P(x) = x^4 - 13x^2 + 36$

b) $Q(x) = x^4 - 34x^2 + 225$

c) $Q(x) = x^4 - 5x^2 + 6$

d) $P(x) = x^5 - x^4 - 26x^3 + 26x^2 + 25x - 25$

e) $P(x) = x^5 - x^4 + 8x^3 - 8x^2 - 9x + 9$

f) $P(x) = x^5 - 3x^4 + 3x^3 - 9x^2 + 2x - 6$

Exercise 18: Simplify the following algebraic fractions:

a) $\dfrac{x+4}{x^2+x-12} =$

b) $\dfrac{9x^2-4}{6x-4} =$

c) $\dfrac{x^3-10x^2+25x}{x^2-6x+5} =$

d) $\dfrac{x^3+x^2-9x-9}{x^2+6x+9} =$

e) $\dfrac{x^2+7x+6}{x^2-14x+49} =$

Exercise 19: Simplify the following algebraic fractions:

a) $\dfrac{x^2yz+xy^2z+xyz^2}{x^3y^3z+x^2y^4z+x^2y^3z^2} =$

b) $\dfrac{a^2bc+ab^2c+abc^2}{a^2b^2c^2} =$

c) $\dfrac{x^3-5x^2+7x-3}{x^2-4x+3} =$

d) $\dfrac{x^7-5x^5+4x^3}{x^7-4x^6+5x^5-2x^4} =$

e) $\dfrac{x^5-5x^4-10x^3+80x^2-96x}{x^2(5x^2-10x)(x^2-16)} =$

Exercise 20: Work out:

a) $\dfrac{x-2}{x+3} - \dfrac{x+1}{x-2} =$

b) $\dfrac{5x+3}{x-5} - \dfrac{x-4}{x} =$

c) $\dfrac{2x-1}{x+3} - \dfrac{x-3}{2x-1} =$

d) $\dfrac{x^2-1}{x+2} - \dfrac{x^2-2}{x+1} =$

Exercise 21: Work out and simplify:

a) $\dfrac{9x-2}{x+5} - \dfrac{2x-7}{x^2-25} =$

b) $\dfrac{x-2}{x^2+6x+5} - \dfrac{2x-3}{x^2-4x-5} =$

c) $\dfrac{3}{x+5} - \dfrac{4x-2}{x^2+7x+10} - \dfrac{x}{x+2} =$

d) $\dfrac{x}{x+2} - \dfrac{5}{x^2-4} + \dfrac{3x}{4-x^2} =$

Exercise 22: Work out and simplify:

a) $\dfrac{5}{x^2-3x-10} + \dfrac{x}{x^2-25} - \dfrac{2}{x+2} =$

b) $\dfrac{2x+7}{x^3+3x^2-x-3} - \dfrac{x-1}{x^2+4x+3} =$

c) $\dfrac{2}{x^2-25} - \dfrac{4x}{5-x} + \dfrac{5}{25-x^2} + \dfrac{9x}{x-5} =$

d) $\dfrac{8}{4-x} - \dfrac{5x-1}{x^2-16} + \dfrac{2x}{x^2-8x+16} =$

Exercise 23: Work out and simplify:

a) $\dfrac{x^2+14x+49}{x^2-6x+9} \cdot \dfrac{x^2-9}{x^2-49} =$

b) $\dfrac{x^3-5x^2}{x^2-4} \cdot \dfrac{3x+6}{2x^2-10x} =$

c) $\dfrac{x^4+x^2}{x-3} : \dfrac{x^3}{x^2-9} =$

d) $\dfrac{7x-49}{x^2+7x} : \dfrac{7}{x^2-49} =$