UNIT 4: CHARACTERISTICS OF FUNCTIONS. TENDENCY

Exercise 1: Find the domain of the following functions:

a) $f(x) = 7x^4 - 5x^3 + 6x^2 - x + 2$

b) $f(x) = \dfrac{7x+3}{x^2+5x+6}$

c) $f(x) = \dfrac{x^5 + 5x^3 - 3x + 4}{x^3 - 2x^2 + x}$

d) $f(x) = \sqrt[3]{x^2 - 4}$

e) $f(x) = \sqrt{x^2 - 2x - 3}$

f) $f(x) = \dfrac{\sqrt{x-2}}{x^2 - x}$

Exercise 2: Find the domain of the following functions:

a) $f(x) = \dfrac{x^4 - 7x^3 + 5x^2 + 1}{x^2 - 16}$

b) $f(x) = \dfrac{7x}{6x - 3}$

c) $f(x) = \dfrac{x^2 - 4x + 3}{x^2 + 1}$

d) $f(x) = \dfrac{\sqrt{x-1}}{x^2 - 4}$

e) $f(x) = \dfrac{1}{\sqrt{x^2 - 5x + 6}}$

f) $f(x) = \dfrac{5x - 10}{\sqrt{1 - x^2}}$

Exercise 3: Find the domain of the following functions:

a) $f(x) = \dfrac{\sqrt{3x+1}}{x^2 - 4}$
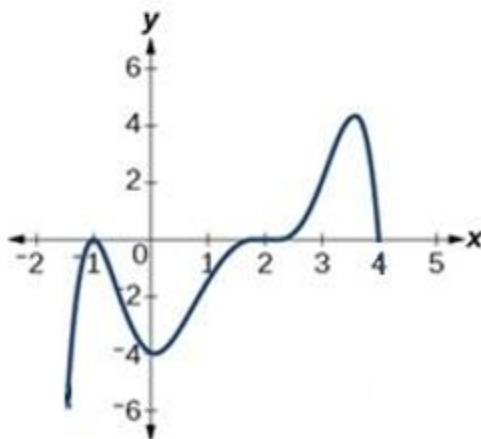
b) $f(x) = \dfrac{\sqrt{x-7}}{x^2 - 9}$
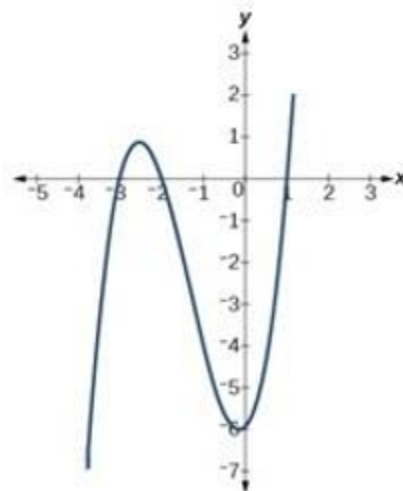
c) $f(x) = \dfrac{x}{\sqrt{x^2 - 16}}$

d) $f(x) = \dfrac{\sqrt{2x+5}}{\sqrt{x^2 - 1}}$

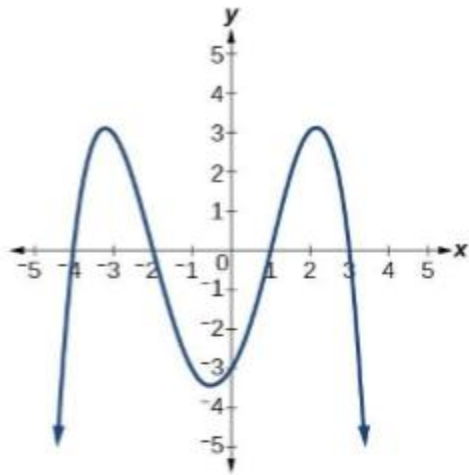**Exercise 4:** Find the domain and the range of the following functions and indicate the points where they cross the axes.
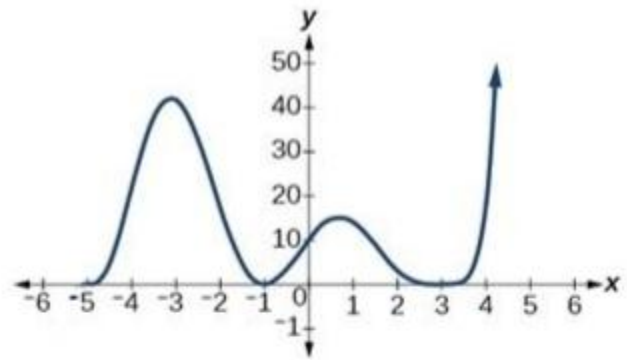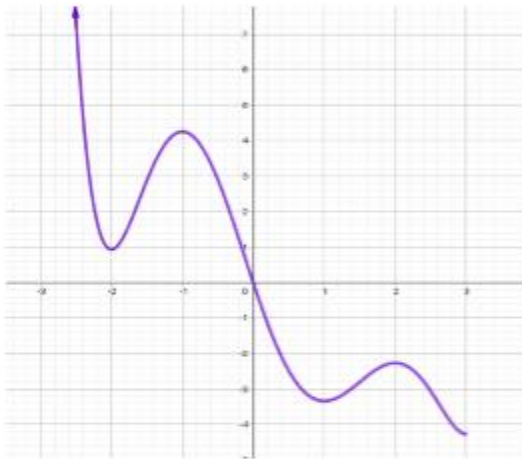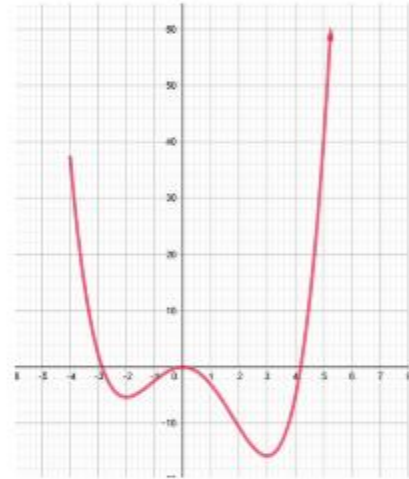
a)

b)

c)

d)

Exercise 5: Study the monotony and the extrema of the following functions:

a)

b)

c)

d)

Exercise 6: Given the following graphs:
   a) Study the domain and the range
   b) Find the points where the function crosses the axes
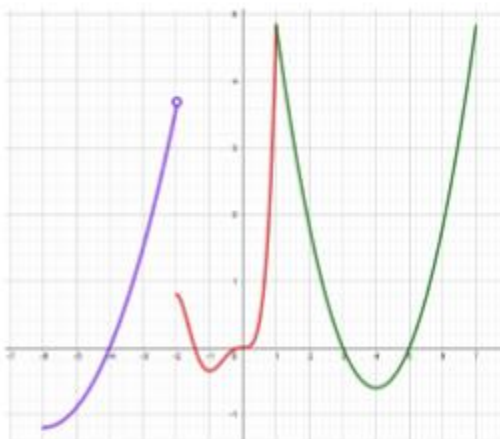   c) Study the monotony
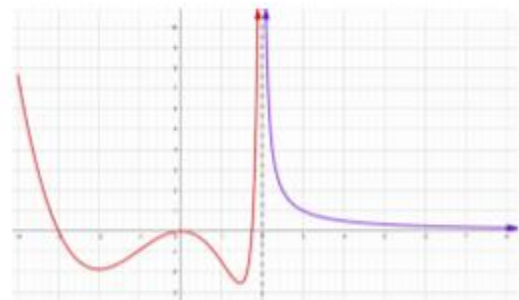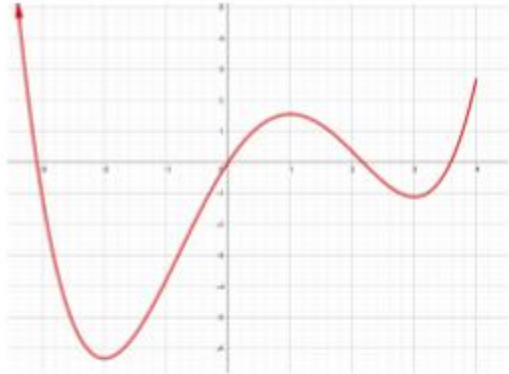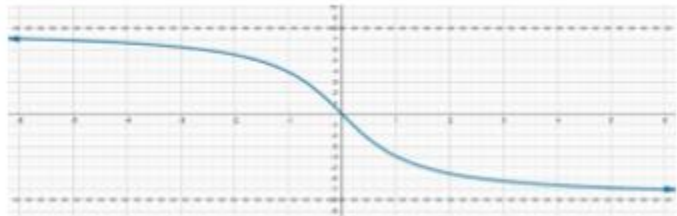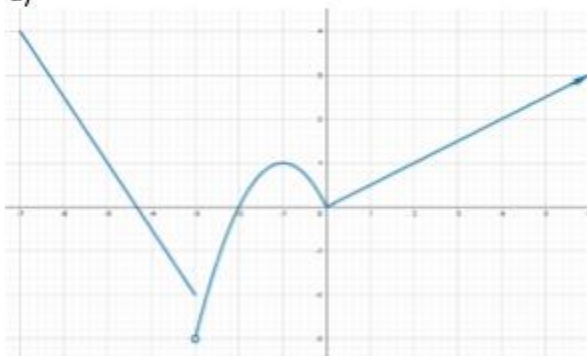   d) Find the local and global extrema

a)

b)

Exercise 7: Given the following graphs:
   a) Study the domain and the range
   b) Find the points where the function crosses the axes
   c) Study the monotony
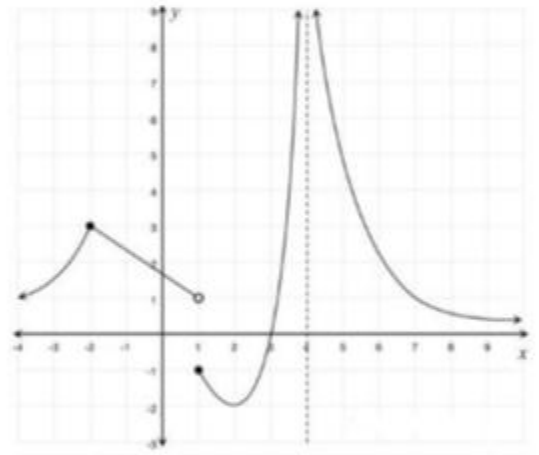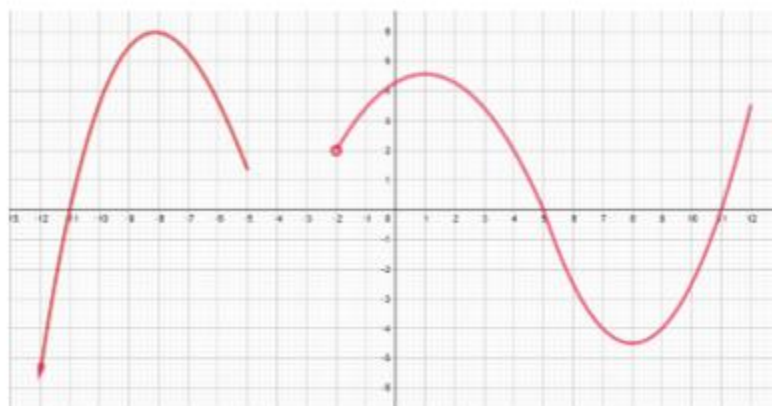   d) Find the local and global extrema
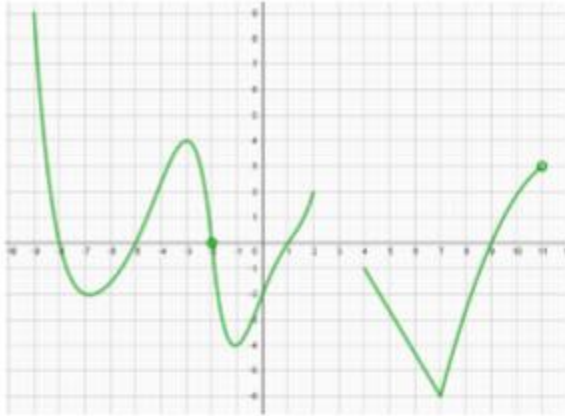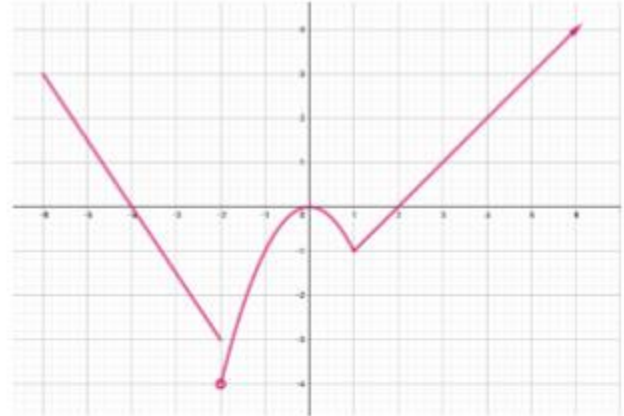   e) Find $f(1)$

a)

b)

c)

d)

e)

Exercise 8: Work out the graph of a function that fulfills all the following characteristics at the same time:
    a) Its domain is $(-5,-2]\cup[3,+\infty)$
    b) It crosses the axes at the points (-4,0) and (3,0)
    c) It has a minimum at $x=-4$ and a maximum at $x=5$, either local or global

Exercise 9: Plot the graph of a function that fulfills all the following characteristics at the same time:
    a) Its domain is $(-5,2]\cup[4,7)$
    b) It crosses the axes at the points $(-2,0)$ and $(0,5)$
    c) It has minima at x = -5 and x = 3 and a maximum at x = 6, either local or global

Exercise 10: Given the following graph of a certain function:
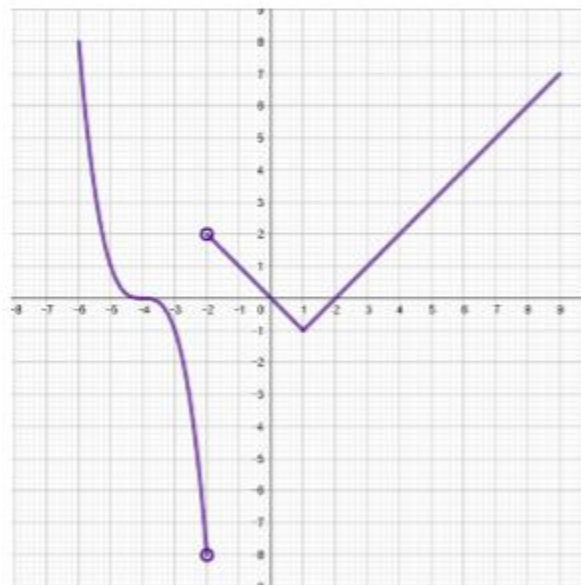
    a) Indicate its domain and its image
    b) Determine the points where the function crosses the axes
    c) Study its monotony
    d) Study the local and global extrema
    e) Find $f(1)$, $f(0)$, $f(9)$, $f(-2)$

Exercise 11: Indicate the asymptotes of the following functions and write the equations of the horizontal and vertical ones. Indicate also the general form of the function:

a)
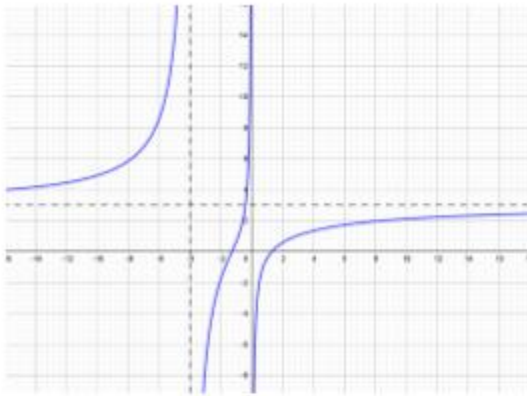
b)
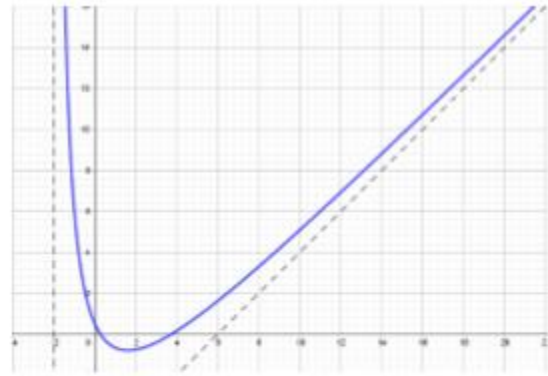
c)

Exercise 12: Given the following graph of a certain function:

a) Find $\lim\limits_{x \to -1^-} f(x)$, $\lim\limits_{x \to -1^+} f(x)$, $f(-1)$

b) Find $\lim\limits_{x \to -1} f(x)$

c) Find $\lim\limits_{x \to 3^-} f(x)$, $\lim\limits_{x \to 3^+} f(x)$, $f(3)$

d) Find $\lim\limits_{x \to 3} f(x)$

Exercise 13: Find the value of the following limits:

a) $\lim\limits_{x\to 2} \dfrac{2x+3}{x+1} =$

b) $\lim\limits_{x\to -1} (x^3 - x^2 + x - 1) =$

c) $\lim\limits_{x\to 5} \left(\sqrt{4x+5} - x\right) =$

d) $\lim\limits_{x\to -\infty} (x^4 - 7x^2 + 1) =$
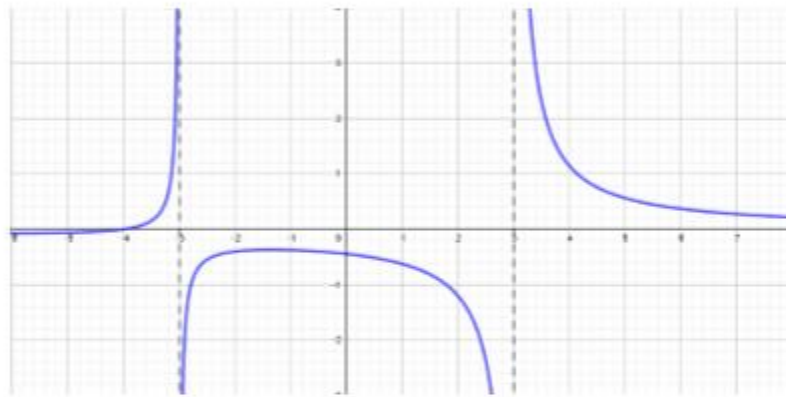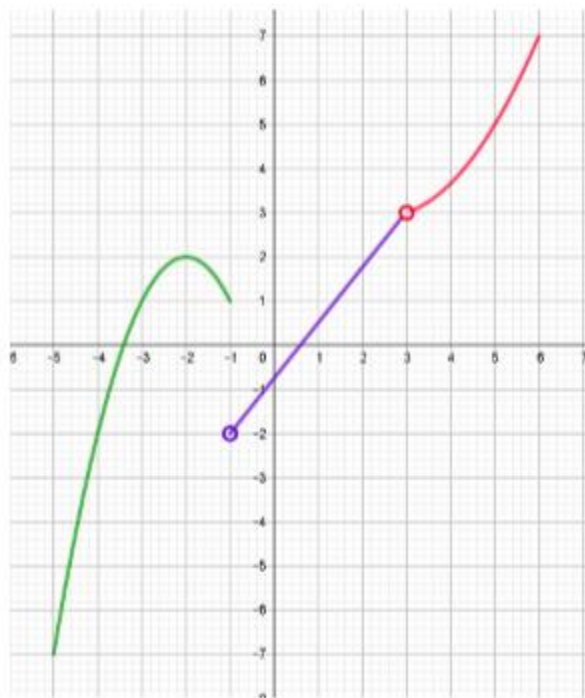
e) $\lim\limits_{x\to -\infty} (-3x^5 + 2x - 4) =$

f) $\lim\limits_{x\to -\infty} (x^3 + 8x^2 - 13x + 25) =$

Exercise 14: Find the value of the following limits:

a) $\lim\limits_{x\to +\infty} \dfrac{2x}{3x^2 - 5x + 2} =$

b) $\lim\limits_{x\to +\infty} \dfrac{2x^2 - 3x + 5}{2x^2 - 3} =$

c) $\lim\limits_{x\to +\infty} \dfrac{3x+1}{1-x} =$

d) $\lim\limits_{n\to +\infty} \dfrac{3n+5}{2\sqrt{n^2 + 2n}} =$

e) $\lim\limits_{x\to +\infty} \dfrac{6x^3}{(x+1)(x+2)(2x-5)} =$

f) $\lim\limits_{x\to +\infty} \dfrac{\sqrt{x^3}}{7x-2} =$

g) $\lim\limits_{x\to +\infty} \dfrac{x^3 + 9x^2 - 7}{x^2 - 4x} =$

h) $\lim\limits_{x\to +\infty} \dfrac{7x^4 - 5x^3}{x^7 - 2x^5 + 7x^2 - 8} =$

i) $\lim\limits_{x\to -\infty} \dfrac{x^2 + 5x + 1}{2x - 1} =$

j) $\lim\limits_{n\to +\infty} \dfrac{5n+9}{\sqrt{9n^2 - 4n} + 2\sqrt{n^2 + 1}} =$

Exercise 15: Find the value of the following limits:

a) $\lim\limits_{x\to 1} \dfrac{x^3 + 3}{x^2 - 1} =$

b) $\lim\limits_{x\to 2} \dfrac{x^2}{x - 2} =$

c) $\lim\limits_{x\to 0} \dfrac{x^2 - 1}{x^2 + 2x} =$

d) $\lim\limits_{x\to 1} \dfrac{3x-5}{x^2 - 2x + 1} =$

Exercise 16: Find the value of the following limits:

a) $\lim\limits_{x\to -3} \dfrac{x+3}{x^2 + x - 6} =$

b) $\lim\limits_{x\to 2} \dfrac{x^2 - 4}{x^3 - x^2 - 2x} =$

c) $\lim\limits_{x\to 5} \dfrac{x^2 - 25}{x - 5} =$

d) $\lim\limits_{x\to 1} \dfrac{x-1}{x^2 - 1} =$

Exercise 17: Find the value of the following limits:

a) $\lim\limits_{x\to 1} \dfrac{x^2 + x - 2}{x^4 + x^2 + x - 3} =$

b) $\lim\limits_{x\to 3} \dfrac{x^2 - 5x + 6}{x^2 - 6x + 9} =$

c) $\lim\limits_{x\to 2} \dfrac{x^4 - 5x^3 + 6x^2 + 4x - 8}{x^3 - 4x^2 + 4x} =$

d) $\lim\limits_{x\to 2} \dfrac{x^4 - 3x^3 + 4x}{x^3 - 7x^2 + 16x - 12} =$

Exercise 18: Find the value of the following limits:

a) $\lim\limits_{x \to +\infty} \left(2x - \dfrac{2x^2 - 3}{x+1} \right) =$

b) $\lim\limits_{x \to +\infty} \left(2x - \dfrac{4x^2 - 10x}{2x+3} \right) =$

c) $\lim\limits_{x \to +\infty} \left(\dfrac{x^2 + 7}{x+2} - \dfrac{x^2 - 5}{x-2} \right) =$

d) $\lim\limits_{x \to 3} \left(\dfrac{x}{x-3} - \dfrac{x^2 - 3x + 3}{x^2 - 5x + 6} \right) =$

Exercise 19: Find the value of the following limits:

a) $\lim\limits_{n \to +\infty} \left(\sqrt{n^2 - n} - n \right) =$

b) $\lim\limits_{n \to +\infty} \left(\sqrt{n+1} - \sqrt{n-1} \right) =$

c) $\lim\limits_{n \to +\infty} \left(\sqrt{n^2 - 3} - n \right) =$

d) $\lim\limits_{n \to +\infty} \left(\sqrt{4n^2 - 3n} - 2n \right) =$

e) $\lim\limits_{n \to +\infty} \left(\sqrt{n^2 - 2n} - \sqrt{n^2 + 4} \right) =$

Exercise 20: Find the value of the following limits:

a) $\lim\limits_{n \to \infty} \left(\dfrac{4n^2 - 1}{2n} - \dfrac{8n + 5}{4} \right) =$

b) $\lim\limits_{n \to \infty} \left(\dfrac{4n - 2}{2} - \dfrac{6n^2 - 3n + 7}{3n} \right) =$

Exercise 21: Find the value of the following limits:

a) $\lim\limits_{x \to 4} \dfrac{x - 4}{\sqrt{x - 2}} =$

b) $\lim\limits_{x \to 1} \dfrac{\sqrt{2x - 1} - 1}{x - 1} =$

Exercise 22: Indicate the horizontal and vertical asymptotes of the functions:

a) $f(x) = \dfrac{7x + 1}{x + 3}$

b) $f(x) = \dfrac{-5x^2 + 1}{x^2 - 2x - 3}$

c) $f(x) = \dfrac{x^3 - 1}{x^2 - 9}$

d) $f(x) = \dfrac{2}{x + 5}$