

UNIT 3: EQUATIONS, INEQUALITIES AND SYSTEMS

Exercise 1: Solve and factorize the following quadratic equations:

a) $x^2 + 64 = 0$

b) $9x^2 - 49 = 0$

c) $x^2 + 7x = 0$

d) $6x^2 - 3 + 4x + 1 = x - 2$

e) $x^2 - 4x - 5 = 0$

f) $x^2 + 12x + 36 = 0$

g) $2x^2 - 1 = 0$

h) $7x^2 - 9 = 0$

Exercise 2: Solve the equation $x(x-1)(x+2)(x-3)^2 = 0$

Exercise 3: Solve:

a) $(x+5)^2 = 0$

b) $(5x-2)^2 = 0$

c) $(x+3)(x-2) = 0$

d) $(2x+1)(x-7) = 0$

Exercise 4: Solve:

a) $(3x-1)^2 = 25$

b) $\frac{(x+3)^2}{25} = \frac{(x-5)^2}{9}$

c) $(x+7)^2 + 6x = -2$

d) $(x-1)^2 - (2x+4)^2 = 5-x$

e) $\frac{x}{4} = \frac{x^2+2}{7x+2}$

f) $\frac{x+5}{2x-2} = \frac{x-1}{x-2}$

Exercise 5: Solve and factorize:

a) $x^4 - 13x^2 + 36 = 0$

b) $x^4 - 11x^2 + 18 = 0$

c) $x^4 - 3x^2 - 4 = 0$

d) $x^4 - 16 = 0$

e) $x^4 - 9x^2 = 0$

f) $x^4 - 10x^2 + 25 = 0$

Exercise 6: Solve the following radical equations:

a) $\sqrt{5x-1} = 6$

b) $2\sqrt{x} - x = -3$

c) $\sqrt{3x+12} + x = 2$

d) $\sqrt{1-4x} + x = 1$

e) $\sqrt{9x+4} - 2 = x$

Exercise 7: Solve the following radical equations:

a) $\sqrt{3x+4} - \sqrt{2x+1} = 1$

b) $\sqrt{3x+1} - \sqrt{x-1} = 2$

c) $\sqrt{2x+5} - \sqrt{3x-2} = 1$

d) $\sqrt{2x-3} - \sqrt{x+7} = 2$

Exercise 8: Solve the following radical equations:

a) $\sqrt{1-3x} + \sqrt{x+6} = 5$

b) $\sqrt{x+1} + \sqrt{1-3x} = 2$

c) $\sqrt{2x+4} - \sqrt{3x+1} = 3$

d) $\sqrt{x+2} + \sqrt{2x+2} = 7$

Exercise 9: Solve and classify the following simultaneous equations using the indicated method:

$$\text{a) } \begin{cases} 3x - 5y = 4 \\ 2x - 3y = 2 \end{cases} \quad \text{Elimination}$$

$$\text{b) } \begin{cases} x - 3y = 7 \\ 2x = 5 + 6y \end{cases} \quad \text{Substitution}$$

$$\text{c) } \begin{cases} 3x + 5y = 17 \\ 2x - y = -6 \end{cases}$$

$$\text{d) } \begin{cases} 6x + 3y = 9 \\ 4x + 2y = 6 \end{cases}$$

$$\text{e) } \begin{cases} x + 2y = 6 \\ 2x - y = 7 \end{cases} \quad \text{Graphically}$$

$$\text{f) } \begin{cases} x + 2y = 6 \\ 2x - y = 7 \end{cases} \quad \text{Graphically}$$

Exercise 10: Solve the following systems of nonlinear equations:

$$\text{a) } \begin{cases} 2x + y = 1 \\ x^2 + y^2 = 10 \end{cases}$$

$$\text{b) } \begin{cases} x + y = 2 \\ x^2 + y^2 = 2 \end{cases}$$

$$\text{c) } \begin{cases} x^2 + y^2 = 25 \\ y - x = 1 \end{cases}$$

$$\text{d) } \begin{cases} y = x^2 \\ 3x = y + 2 \end{cases}$$

$$\text{e) } \begin{cases} y^2 = x + 3 \\ 2y = x + 4 \end{cases}$$

$$\text{f) } \begin{cases} x^2 - y^2 = 16 \\ x - 2y = 1 \end{cases}$$

$$\text{g) } \begin{cases} 3x + y = 9 \\ x^2 - y^2 = -5 \end{cases}$$

$$\text{h) } \begin{cases} x + 2y = 6 \\ x^2 + 4y^2 = 20 \end{cases}$$

$$\text{i) } \begin{cases} 2x + y = 3 \\ x^2 + y^2 = 18 \end{cases}$$

$$\text{j) } \begin{cases} 2x - y = 3 \\ x^2 - y = 3 \end{cases}$$

$$\text{k) } \begin{cases} xy = 2 \\ x^2 - 2y^2 = 2 \end{cases}$$

$$\text{l) } \begin{cases} x^2 + y^2 = 13 \\ x^2 - y^2 = 5 \end{cases}$$

$$\text{m) } \begin{cases} x^2 + y^2 = 10 \\ x^2 - y^2 = -12 \end{cases}$$

$$\text{n) } \begin{cases} x^2 - y = 5 \\ x^2 + y^2 = 25 \end{cases}$$

$$\text{o) } \begin{cases} xy = 8 \\ x^2 + y^2 = 20 \end{cases}$$

$$\text{p) } \begin{cases} xy = -2 \\ x^2 + y^2 = 5 \end{cases}$$

Exercise 11: Find the dimensions of a rectangle if its perimeter has a length of 60 m and its area measures 221 m²

Exercise 12: Find two numbers so that their product is 8 and the sum of their squares is 20

Exercise 13: Solve the following linear inequalities:

$$\text{a) } 3x - 8 < 5x + 3$$

$$\text{b) } 2(x - 5) - 3(x - 2) \leq 0$$

$$\text{c) } 1 - (2x - 3) > 5x - 8$$

$$\text{d) } 3x - 5(x - 4) < 7 - 2x$$

Exercise 14: Solve the following second degree inequalities:

a) $(x-2)(x+3) > 0$

b) $x^2 + 9x + 8 \geq 0$

c) $x^2 - 25 < 0$

d) $(x-7)^2 \leq 0$

Exercise 15: Solve the following second degree inequalities:

a) $x^2 - 10x + 25 < 0$

b) $(x+7)(x-7) - (x-4)(x+4) \geq 0$

c) $2(x-1)^2 - (2x+1)^2 > 1$

d) $\frac{(x+2)(x-2)}{2} + x \leq \frac{3x+2}{2}$

Exercise 16: Solve the following systems of inequalities:

a)
$$\left. \begin{array}{l} 8 - 2(x+3) \leq 4(x-1) \\ 1 - (x-6) > 3x - (3-x) \end{array} \right\}$$

b)
$$\left. \begin{array}{l} x^2 - 4x + 3 \leq 0 \\ x^2 - 9 \geq 0 \end{array} \right\}$$

c)
$$\left. \begin{array}{l} x^2 + 3x - 10 \geq 0 \\ x^2 - 4 < 0 \end{array} \right\}$$

d)
$$\left. \begin{array}{l} x^2 + 5x \leq 0 \\ x^2 - 4 > 0 \end{array} \right\}$$

Exercise 17: Solve the following systems of inequalities:

a)
$$\left. \begin{array}{l} 3(x-4) - 2(x+5) \leq 3x - 7(2x-3) \\ x^2 - 2x \geq 0 \end{array} \right\}$$

b)
$$\left. \begin{array}{l} x^2 - 6x + 9 > 0 \\ x^2 - 16 < 0 \end{array} \right\}$$

c)
$$\left. \begin{array}{l} x^3 - x^2 - 2x \geq 0 \\ x^2 - 4 < 0 \end{array} \right\}$$

d)
$$\left. \begin{array}{l} x^2 + 2x - 15 > 0 \\ x^2 + 4x - 12 \leq 0 \end{array} \right\}$$

Exercise 18: Solve the following systems of inequalities:

a)
$$\left. \begin{array}{l} x^2 + 8x + 7 \geq 0 \\ x^2 + 10x + 9 \leq 0 \end{array} \right\}$$

b)
$$\left. \begin{array}{l} x^2 - 14x + 49 > 0 \\ x^2 - 49 > 0 \end{array} \right\}$$

c)
$$\left. \begin{array}{l} x^2 - 9 < 0 \\ x^2 - 16 \geq 0 \end{array} \right\}$$

d)
$$\left. \begin{array}{l} x^2 + 1 \geq 0 \\ x^2 + 4x + 4 \leq 0 \end{array} \right\}$$

Exercise 19: Solve the following systems of linear inequalities with two variables:

a)
$$\left. \begin{array}{l} x + 2y \leq 1 \\ 2x - y > 7 \end{array} \right\}$$

b)
$$\left. \begin{array}{l} x + 2y < 0 \\ 3x + y \geq -5 \end{array} \right\}$$

c)
$$\left. \begin{array}{l} 5x - y < 7 \\ x + 7y > 4 \end{array} \right\}$$

d)
$$\left. \begin{array}{l} 2x - y < 0 \\ 3x + y \geq 5 \end{array} \right\}$$

e)
$$\left. \begin{array}{l} x \geq 1 \\ y \geq 2 \\ x + y < 7 \end{array} \right\}$$