# UNIT 8: ANALYTIC GEOMETRY

**Exercise 1:** Plot

a) Plot the vectors $\vec{u}=(-3,1)$, $\vec{v}=(-5,-7)$, $\vec{w}=(\sqrt{2},-\sqrt{3})$, $\vec{i}=(1,0)$, $\vec{j}=(0,-2)$

b) Plot the vector $\vec{w}=(1,5)$ with initial point $A(3,1)$

**Exercise 2:** Given the points $A(2,-3)$, $B(5,4)$ and $C(-1,-5)$ find the coordinates of the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$

**Exercise 3:** If $\overrightarrow{PQ}=(7,1)$ and $P(2,-1)$, find the coordinates of the point $Q$

**Exercise 4:** The terminal point of a certain vector $\vec{w}=(5,-1)$ is $S(7,-3)$. Find its initial point.

**Exercise 5:** Find the length of the vectors $\vec{u}=(3,-4)$, $\vec{v}=(-2,-9)$ and $\vec{w}=(1,\sqrt{2})$

**Exercise 6:** Find the length of the vectors $\vec{u}=\left(\frac{1}{3},-\frac{2}{5}\right)$, $\vec{v}=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $\vec{w}=(1+\sqrt{3},\sqrt{5})$

**Exercise 7:** Given the vectors $\vec{u}=(-1,1)$, $\vec{v}=(3,-4)$ and $\vec{w}=(-1,-8)$ work out:

a) $\vec{u}+\vec{v}=$

b) $\vec{u}-\vec{w}=$

c) $\vec{u}-\vec{v}+\vec{w}=$

d) $3\vec{u}-5\vec{v}=$

**Exercise 8:** Given the vectors $\vec{u}=(4,-7)$, $\vec{v}=(-2,1)$ and $\vec{w}=(16,-23)$, express $\vec{w}$ as a linear combination of $\vec{u}$ and $\vec{v}$

**Exercise 9:** Given the vectors $\vec{u}=(4,2)$, $\vec{v}=(-6,3)$ and $\vec{w}=(2,0)$, check if $\vec{w}$ can be expressed as a linear combination of $\vec{u}$ and $\vec{v}$

**Exercise 10:** Write $\vec{w}=(4,-2)$ as a linear combination of $\vec{u}=(-1,3)$ and $\vec{v}=(5,-2)$

**Exercise 11:** Find the dot product of the vectors $\vec{u}=(-3,-1)$ and $\vec{v}=(4,-5)$

**Exercise 12:** Find two vectors that are perpendicular to $\vec{u}=(1,-3)$

**Exercise 13:** Given the vectors $\vec{u}=(3,-7)$, $\vec{v}=(2,-6)$ and $\vec{w}=(7,3)$

a) Are $\vec{u}$ and $\vec{v}$ perpendicular vectors?

b) Are $\vec{u}$ and $\vec{w}$ orthogonal vectors?

c) Express $\vec{v}$ as a linear combination of $\vec{u}$ and $\vec{w}$

**Exercise 14:** Find the value of $k$ so that the vectors $\vec{u}=(k-2,7)$ and $\vec{v}=(k+2,2k+7)$ are orthogonal

**Exercise 15:** Find the value of *k* so that the points $A(2,k+1)$, $B(k,5)$ and $C(1,1)$ form a right-angled triangle.

**Exercise 16:** Prove that the triangle formed by the points $A(3,5)$, $B(10,0)$ and $C(4,-1)$ is isosceles

**Exercise 17:** Find the value of *k* so that the triangle formed by the points $A(k,5)$, $B(3k,6)$ and $C(k-1,k+8)$ is isosceles

**Exercise 18:** If $\vec{u}=(2,-1)$ and $\vec{v}=(3,5)$ find a third vector $\vec{w}$ so that $\vec{w}\cdot\vec{u}=1$ and $\vec{w}\perp\vec{v}$

**Exercise 19:** Given the vectors $\vec{u}=(3,2)$, $\vec{v}=(\sqrt{3},\sqrt{2})$, $\vec{w}=(4,-6)$ and $\vec{z}=(5,-1)$

a) Find the magnitude (length) of the vector $\vec{v}$
b) Express $\vec{w}$ as a linear combination of $\vec{u}$ and $\vec{z}$
c) Are $\vec{u}$ and $\vec{z}$ perpendicular vectors?
d) Indicate the coordinates of the vector $\vec{u}+3\vec{w}-2\vec{z}$

**Exercise 20:** Given the points $A(-1,4)$ and $B(3,-1)$

a) Find the direction vector of the line that goes through $A$ and $B$
b) Find the vector equation of the line $AB$
c) Find the parametric equations of $AB$
d) Write the continuous and general equations

**Exercise 21:** Given the straight line $r\equiv\begin{cases}x=1-3t\\y=-2+5t\end{cases}$ find the direction vector and a point of the line

**Exercise 22:** Find the parametric and continuous equations of the straight line $5x-y+1=0$

**Exercise 23:** Find the value of *k* so that the point $R(k,-2)$ belongs to the straight line

$$r\equiv\begin{cases}x=2-3t\\y=-1+4t\end{cases}$$

**Exercise 24:** Given the points $A(1,-5)$ and $B(-2,7)$ find the continuous and general equations of the straight line that goes through them

**Exercise 25:** Given the straight line

$$r \equiv \begin{cases} x = 2 + 5t \\ y = t - 4 \end{cases}$$

a) Find a point $Q$ that belongs to $r$

b) Decide if the point $P(4,-3) \in r$

c) Find the value ok $k$ so that $A(k+1,7) \in r$

**Exercise 26:** Given the straight line $\frac{x+1}{5} = \frac{4-y}{2}$

a) Find a point on the line

b) Find the direction vector

c) Write the parametric equations

**Exercise 27:** Find the vector, continuous and parametric equations of the straight line $7x + 4y - 9 = 0$

**Exercise 28:** One line passes through the points $A(-1,-2)$ and $B(1,2)$; another line passes through the points $P(-2,0)$ and $Q(0,4)$. Are these lines parallel, perpendicular, or neither?

**Exercise 29:** One line passes through the points $A(0,-4)$ and $B(-1,-7)$; another line passes through the points $P(3,0)$ and $Q(-3,2)$. Are these lines parallel, perpendicular, or neither?

**Exercise 30:** Given the straight line $3x + 2y - 5 = 0$

a) Find the equation of a parallel line that goes through $A(-2,4)$

b) Find the equation of a perpendicular line that goes through $B(3,-5)$

**Exercise 31:** Given the straight line $r \equiv \begin{cases} x = -1 + 2t \\ y = 5 - t \end{cases}$ find the general equations of:

a) A straight line parallel to $r$ going through $Q(-2,3)$

b) A line perpendicular to $r$ going through $R(-5,-1)$

**Exercise 32:** Find the equations of two straight lines $s_1$ and $s_2$ so they both go through the point $A(5,-1)$, $s_1 \parallel r$, $s_2 \perp r$, where $r \equiv x + 2y - 7 = 0$

**Exercise 33:** Given the straight line

$$r \equiv \begin{cases} 4 + 3t \\ 2t - 1 \end{cases}$$

a) Find the general equation of a parallel line r' that passes through the point $A(-2,5)$

b) Find the general equation a perpendicular line r'' that passes through the point $B(-4,1)$

c) Find the point where r and r'' cross

**Exercise 34:** Find the points that divide the segment $\overline{AB}$ in four equal parts, where $A(-6,5)$ and $B(8,-1)$

**Exercise 35:** Given the points $P(1,k+3)$, $Q(k-6,2)$ and $R(-k,-2k)$

a) Find the value of $k$ so that they are aligned

b) Find the value of $k$ (another) so that the triangle $PQR$ is isosceles

**Exercise 36:** Find the coordinates of the symmetric point of $A(2,-5)$ with respect to $Q(-2,9)$

**Exercise 37:** Find the value of $k$ so that $d(P,Q)=d(P,R)$, where $P(7,-2k)$, $Q(-k,k+3)$, $R(6-2k,-1)$

**Exercise 38:**

a) Determine if the points $A(3,6)$, $B(-3,2)$ and $C(0,4)$ are aligned. If the answer is yes, find the continuous equation of the straight line they belong to.

b) Work out the coordinates of the symmetric point of $P(3,1)$ with respect to $Q(-3,7)$

c) Find the value of *k* so that the point $R(k,-2)$ belongs to the straight line

$$r \equiv \begin{cases} 2-3t \\ -1+4t \end{cases}$$

**Exercise 39:** The points $A(1,1)$, $B(5,4)$ and $C(5,-1)$ are the vertices of a triangle

a) Find the altitude of the triangle if the base is $\overline{AC}$

b) Find its perimeter and its area

**Exercise 40:** Find the value of $k$ so that $\vec{u}=(-2,k)$ is parallel to $\vec{v}=(1-k,3)$

**Exercise 41:** Find the value of $k$ so that $\vec{u}=(k-2,-7)$ is orthogonal to $\vec{v}=(k+2,2k-7)$

**Exercise 42:** Find the value of $k$ so that $r \perp s$, where $r \equiv kx+2y-3=0$, $s \equiv 5x-ky+1=0$

**Exercise 43:** Find the value of $k$ so that the triangle $A(5,3)$, $B(k+1,4)$ and $C(2k,5)$

a) Has a right angle in $A$

b) Is isosceles